Integral Calculus Formula Sheet

Derivative Rules:

$\frac{d}{dx}(c)=0$	$\frac{d}{dx}(x^n)=nx^{n-1}$	
$\frac{d}{dx}(\sin x)=\cos x$ $\frac{d}{dx}(\sec x)=\sec x\tan x$ $\frac{d}{dx}(\tan x)=\sec^2 x$	$\frac{d}{dx}(\cos x)=-\sin x$ $\frac{d}{dx}(\csc x)=-\csc x\cot x$ $\frac{d}{dx}(\cot x)=-\csc^2 x$	$\frac{d}{dx}(a^x)=a^x\ln a$ $\frac{d}{dx}(e^x)=e^x$
$\frac{d}{dx}(cf(x))=c\frac{d}{dx}(f(x))$	$\frac{d}{dx}(f(x)\pm g(x))=\frac{d}{dx}(f(x))\pm\frac{d}{dx}(g(x))$	
$(f\cdot g)'=f'\cdot g+f\cdot g'$	$\left(\frac{f}{g}\right)'=\frac{f'g-fg'}{g^2}$	$\frac{d}{dx}(f(g(x)))=f'(g(x))g'(x)$

Properties of Integrals:

$\int kf(u)du=k\int f(u)du$	$\int[f(u)\pm g(u)]du=\int f(u)du\pm\int g(u)du$
$\int_a^a f(x)dx=0$	$\int_a^b f(x)dx=-\int_b^a f(x)dx$
$\int_a^c f(x)dx=\int_a^b f(x)dx+\int_b^c f(x)dx$	$f_{ave}=\frac{1}{b-a}\int_a^b f(x)dx$
$\int_{-a}^a f(x)dx=2\int_0^a f(x)dx$ if f(x) is even	$\int_{-a}^a f(x)dx=0$ if f(x) is odd
$\int_a^b g(f(x))f'(x)dx=\int_{f(a)}^{f(b)} g(u)du$	$\int udv=uv-\int vdu$

Integration Rules:

$\int du=u+C$ $\int u^n du=\frac{u^{n+1}}{n+1}+C$ $\int\frac{du}{u}=\ln\lvert u\rvert+C$ $\int e^u du=e^u+C$ $\int a^u du=\frac{1}{\ln a}a^u+C$	$\int\sin u\,du=-\cos u+C$ $\int\cos u\,du=\sin u+C$ $\int\sec^2 u\,du=\tan u+C$ $\int\csc^2 u=-\cot u+C$ $\int\csc u\cot u\,du=-\csc u+C$ $\int\sec u\tan u\,du=\sec u+C$	$\int\frac{du}{a^2+u^2}=\frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$ $\int\frac{du}{\sqrt{a^2-u^2}}=\arcsin\left(\frac{u}{a}\right)+C$ $\int\frac{du}{u\sqrt{u^2-a^2}}=\frac{1}{a}arc\sec\left(\frac{\lvert u\rvert}{a}\right)+C$

Fundamental Theorem of Calculus:

$F'(x)=\frac{d}{dx}\int_a^x f(t)\,dt=f(x)$ where $f(t)$ is a continuous function on [*a, x*].
$\int_a^b f(x)\,dx=F(b)-F(a)$, where F(x) is any antiderivative of f(x).

Riemann Sums:

$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$ $\sum_{i=1}^{n} a_i + b_i = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$	$\int_a^b f(x)dx=\lim_{n\to\infty}\sum_{i=1}^{n} f(a+i\Delta x)\Delta x$ $\Delta x=\frac{b-a}{n}$
$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$	$\sum_i (\text{height of } i\text{th rectangle})\cdot(\text{width of } i\text{th rectangle})$ Right Endpoint Rule: $\sum_{i=1}^{n} f(a+i\Delta x)(\Delta x)=\sum_{i=1}^{n}(\frac{(b-a)}{n})f(a+i\frac{(b-a)}{n})$ Left Endpoint Rule: $\sum_{i=1}^{n} f(a+(i-1)\Delta x)(\Delta x)=\sum_{i=1}^{n}(\frac{(b-a)}{n})f(a+(i-1)\frac{(b-a)}{n})$ Midpoint Rule: $\sum_{i=1}^{n} f(a+\left(\frac{(i-1)+i}{2}\right)\Delta x)(\Delta x)=\sum_{i=1}^{n}(\frac{(b-a)}{n})f(a+\left(\frac{(i-1)+i}{2}\right)\frac{(b-a)}{n})$

Net Change:

Displacement: $\int_a^b v(x)dx$	Distance Traveled: $\int_a^b \lvert v(x)\rvert dx$	$s(t)=s(0)+\int_0^t v(x)dx$	$Q(t)=Q(0)+\int_0^t Q'(x)dx$

Trig Formulas:

$\sin^2(x)=\frac{1}{2}(1-\cos(2x))$	$\tan x=\frac{\sin x}{\cos x}$	$\sec x=\frac{1}{\cos x}$	$\cos(-x)=\cos(x)$	$\sin^2(x)+\cos^2(x)=1$
$\cos^2(x)=\frac{1}{2}(1+\cos(2x))$	$\cot x=\frac{\cos x}{\sin x}$	$\csc x=\frac{1}{\sin x}$	$\sin(-x)=-\sin(x)$	$\tan^2(x)+1=\sec^2(x)$

Geometry Fomulas:

Area of a Square: $A=s^2$	Area of a Triangle: $A=\frac{1}{2}bh$	Area of an Equilateral Trangle: $A=\frac{\sqrt{3}}{4}s^2$	Area of a Circle: $A=\pi r^2$	Area of a Rectangle: $A=bh$

Areas and Volumes:

Area in terms of *x* (vertical rectangles): $\int_a^b (top - bottom)dx$	Area in terms of *y* (horizontal rectangles): $\int_c^d (right - left)dy$
General Volumes by Slicing: Given: Base and shape of Cross-sections $V = \int_a^b A(x)dx$ if slices are vertical $V = \int_c^d A(y)dy$ if slices are horizontal	Disk Method: For volumes of revolution laying on the axis with slices perpendicular to the axis $V = \int_a^b \pi\left[R(x)\right]^2 dx$ if slices are vertical $V = \int_c^d \pi\left[R(y)\right]^2 dy$ if slices are horizontal
Washer Method: For volumes of revolution not laying on the axis with slices perpendicular to the axis $V = \int_a^b \pi\left[R(x)\right]^2 - \pi\left[r(x)\right]^2 dx$ if slices are vertical $V = \int_c^d \pi\left[R(y)\right]^2 - \pi\left[r(y)\right]^2 dy$ if slices are horizontal	Shell Method: For volumes of revolution with slices parallel to the axis $V = \int_a^b 2\pi rhdx$ if slices are vertical $V = \int_c^d 2\pi rhdy$ if slices are horizontal

Physical Applications:

Physics Formulas	Associated Calculus Problems
Mass: Mass = Density * Volume *(for 3-D objects)* Mass = Density * Area *(for 2-D objects)* Mass = Density * Length *(for 1-D objects)*	Mass of a one-dimensional object with variable linear density: $Mass = \int_a^b (linear\ density) \underbrace{dx}_{distance} = \int_a^b \rho(x)dx$
Work: Work = Force * Distance Work = Mass * Gravity * Distance Work = Volume * Density * Gravity * Distance	Work to stretch or compress a spring (force varies): $Work = \int_a^b (force)dx = \int_a^b F(x)dx = \int_a^b \underbrace{kx}_{\substack{Hooke's\ Law \\ for\ springs}} dx$ Work to lift liquid: $Work = \int_c^d (gravity)(density)(distance)\underbrace{(area\ of\ a\ slice)dy}_{volume}$ $W = \int_c^d 9.8 * \rho * A(y) * (a - y)dy \quad (in\ metric)$
Force/Pressure: Force = Pressure * Area Pressure = Density * Gravity * Depth	Force of water pressure on a vertical surface: $Force = \int_c^d (gravity)(density)(depth)\underbrace{(width)dy}_{area}$ $F = \int_c^d 9.8 * \rho * (a - y) * w(y)dy \quad (in\ metric)$

Integration by Parts:

Knowing which function to call *u* and which to call *dv* takes some practice. Here is a general guide:

u	**I**nverse Trig Function	($\sin^{-1} x, \arccos x,$ etc)
↑	**L**ogarithmic Functions	($\log 3x, \ln(x+1),$ etc)
	Algebraic Functions	($x^3, x+5, 1/x,$ etc)
↓	**T**rig Functions	($\sin(5x), \tan(x),$ etc)
dv	**E**xponential Functions	($e^{3x}, 5^{3x},$ etc)

Functions that appear at the top of the list are more like to be *u,* functions at the bottom of the list are more like to be *dv.*

Trig Integrals:

Integrals involving sin(x) and cos(x):	Integrals involving sec(x) and tan(x):
1. If the power of the sine is odd and positive: **Goal:** $u = \cos x$ i. Save a $du = \sin(x)dx$ ii. Convert the remaining factors to $\cos(x)$ (using $\sin^2 x = 1 - \cos^2 x$.)	1. If the power of $\sec(x)$ is even and positive: **Goal:** $u = \tan x$ i. Save a $du = \sec^2(x)dx$ ii. Convert the remaining factors to $\tan(x)$ (using $\sec^2 x = 1 + \tan^2 x$.)
2. If the power of the cosine is odd and positive: **Goal:** $u = \sin x$ i. Save a $du = \cos(x)dx$ ii. Convert the remaining factors to $\sin(x)$ (using $\cos^2 x = 1 - \sin^2 x$.)	2. If the power of $\tan(x)$ is odd and positive: **Goal:** $u = \sec(x)$ i. Save a $du = \sec(x)\tan(x)dx$ ii. Convert the remaining factors to $\sec(x)$ (using $\sec^2 x - 1 = \tan^2 x$.)
3. If both $\sin(x)$ and $\cos(x)$ have even powers: Use the half angle identities: i. $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ ii. $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$	• If there are no sec(x) factors and the power of tan(x) is even and positive, use $\sec^2 x - 1 = \tan^2 x$ to convert one $\tan^2 x$ to $\sec^2 x$ • Rules for sec(x) and tan(x) also work for csc(x) and cot(x) with appropriate negative signs
If nothing else works, convert everything to sines and cosines.	

Trig Substitution:

Expression	Substitution	Domain	Simplification
$\sqrt{a^2 - u^2}$	$u = a\sin\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$\sqrt{a^2 - u^2} = a\cos\theta$
$\sqrt{a^2 + u^2}$	$u = a\tan\theta$	$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$\sqrt{a^2 + u^2} = a\sec\theta$
$\sqrt{u^2 - a^2}$	$u = a\sec\theta$	$0 \le \theta \le \pi, \theta \ne \frac{\pi}{2}$	$\sqrt{u^2 - a^2} = a\tan\theta$

Partial Fractions:

Linear factors:	Irreducible quadratic factors:
$\frac{P(x)}{(x-r_1)^m} = \frac{A}{(x-r_1)} + \frac{B}{(x-r_1)^2} + \ldots + \frac{Y}{(x-r_1)^{m-1}} + \frac{Z}{(x-r_1)^m}$	$\frac{P(x)}{(x^2+r_1)^m} = \frac{Ax+B}{(x^2+r_1)} + \frac{Cx+D}{(x^2+r_1)^2} + \ldots + \frac{Wx+X}{(x^2+r_1)^{m-1}} + \frac{Yx+Z}{(x^2+r_1)^m}$
If the fraction has multiple factors in the denominator, we just add the decompositions.	